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## INVENTORY MODEL DEPENDING UPON TIME AND PRICE FUNCTION

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**ABSTRACT:** Inventory cost is an important part of the enterprise operation cost. For deteriorating items, especially those with high deteriorating rate, deterioration is a key characteristic and its impact on modeling of inventory systems cannot be neglected. So the deterioration rate should be taken into consideration in the development of inventory strategy. For different kinds of enterprises, the emphasis on the deteriorating items inventory study is different. For the seller of deteriorating items, the current studies can be divided into two types; the first type emphasizes the inventory strategies for the retailer of the deteriorating items, the second type focuses on the inventory policy under a two-warehouse system. For the manufactures of deteriorating items, the current emphasis is on developing an optimal production-inventory strategy.

**KEYWORDS:** Deterioration, strategies

### **INTRODUCTION**

Inventory models for determining the optimal sticking policies for items deteriorating with time have engaged attention of researchers in recent years. A good number of authors have taken different rate of deterioration in their analysis such as constant, linear function of time and two parameter Weibull function etc. as the utility of goods does not remain constant over time. A look at the literature available on inventory reveals that several models have been formulated in a static environment considering demand rates which were constant. This kind of a demand rate had the simple and the only advantage of providing a minimalism to the study. It was at that time also a known fact that there can be no commodity which can boast of a constant demand rate, unaltering in its nature with time and other market forces. Later on the demand rate was assumed to depend upon single factors, like, time, stock, selling price etc. Such kind of assumptions, were no doubt, a vast improvement over the constancy of the demand. They at least, were allowing for some kind of dynamicity in the nature of the demand created by the commodity in the market.

With the advent of supermarkets, it was commonly acknowledged that vast displays of stocks induce the customer into buying more. Also it was noted that a decline in the level of displayed stock witnessed a decline in the customer's demand for that item. The dependence of the sale of any item on its selling price is not a new concept, but a common sense conclusion. It is a general observation that an increase in the selling price of the commodity will deter its customer's from opting that item in future. However, a dip in the selling price, in whatever form it may come, always notices a sudden increase in the demand rate, as a reduction in prices encourages the customers to buy more.

But changing market conditions have rendered the dependence of demand rate on a single factor quite unfruitful, since in real life situation, a demand cannot depend exclusively on a single parameter. It is always observed that a surge or decline in the demand is attributed to a combination of two or more factors. Such a combination of two or more factors grants more authenticity to the formulation of the model and makes it more close to reality. In recent years, mathematical ideas have been used in different areas in real life problems, particularly for controlling inventory. One of the important concerns of the management is to decide when and how much to order or to manufacture so that the total cost associated with the inventory system becomes minimum. This is some what more important, when the inventory under go decay or deterioration. When the items of the commodity are kept in the stock a an inventory for fulfilling the future demand, it has been observed that demand rate of an item may be influenced by the amount of stock level, selling price and other factors that is, the consumption rate may go up or down. It is a well known fact that lesser the selling price increase the demand of that item and vice-versa as selling

price of an item is one of the very important factors for the marketing researchers and practitioners to investigate the modeling aspects of this phenomenon.

## ASSUMPTIONS AND NOTATIONS

i. The demand rate is decreasing with linear price function and decreases negative exponentially with time i.e.  $D(P,t) = d(P)e^{-\alpha t}$  is the demand rate at price 'P' and time 't', where d(P) is the initial positive price dependent demand rate where  $d(P) = \beta - \gamma P$ ,  $\beta$  and  $\gamma$  are positive constants, 'P' is

the unit selling price per unit  $\left(\frac{\beta}{\gamma} > P\right)$ .  $\alpha > 0$  is a constant governing the decreasing rate of the

demand rate.

ii. A variable fraction  $\pi(t)$  of on hand inventory deteriorates per unit time, in this model  $\pi(t)$  is assumed in the form  $\pi(t) = a + bt$ , 0 < a, b << 1, t > 0.

- iii. I (t) is the on hand inventory at any time t.
- iv.  $I_w(t)$  is the on hand inventory without decay at time t.
- v. Z (t) is the stock loss due to decay at time t.
- vi. There is no replacement or repair of the decayed units during the period under consideration.
- vii. C is the unit purchase cost, K is the ordering cost per order, S is the unit shortage cost rate and they are constant during scheduling period T.
- viii. Holding cost  $H = h + \phi t$  per unit.

#### MATHEMATICAL MODELING

The behavior of the inventory system is given by the following differential equation:

$$\frac{\mathrm{d}\mathbf{I}(t)}{\mathrm{d}t} + \pi(t)\mathbf{I}(t) = -\mathbf{D}(\mathbf{P}, t)$$

 $\mathbf{I}(t)\mathbf{e}^{\left(at+\frac{\mathbf{b}t^{2}}{2}\right)} = -\mathbf{d}(\mathbf{P})\mathbf{\int}\mathbf{e}^{\left(at-\alpha t+\frac{\mathbf{b}t^{2}}{2}\right)}\mathbf{d}t + \mathbf{C}_{\mathbf{T}}$ 

Under the assumptions, this equation becomes

$$\frac{\mathrm{d}\mathbf{I}(\mathbf{t})}{\mathrm{d}\mathbf{t}} + (\mathbf{a} + \mathbf{b}\mathbf{t})\mathbf{I}(\mathbf{t}) = -\mathbf{d}(\mathbf{P})\mathbf{e}^{-\alpha t}.$$
 ... (1.1)

The solution of equation (1.1) is given by

$$\mathbf{I}(t) \mathbf{e}^{\left(at + \frac{bt^2}{2}\right)} = \int -d(\mathbf{P}) \mathbf{e}^{\alpha t} \cdot \mathbf{e}^{\left(at + \frac{bt^2}{2}\right)} dt + C_{\mathbf{I}}$$

where  $C_I$  is constant of integration.

or

0

$$= -d(P)\int \left(1 + at - \alpha t + \frac{bt^{2}}{2}\right)dt + C_{I}$$
  
r 
$$I(t) = -d(P)e^{-\left(at + \frac{bt^{2}}{2}\right)}\left[t + (a - \alpha)\frac{t^{2}}{2} + \frac{bt^{3}}{6}\right] + C_{I}e^{-\left(at + \frac{bt^{2}}{2}\right)}.$$

At  $t = 0, C_I = I(0)$ 

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$$\Rightarrow I(t) = -d(P)e^{-\left(at + \frac{bt^2}{2}\right)} \left[t + \frac{(a - \alpha)t^2}{2} + \frac{bt^3}{6}\right] + I(0)e^{-\left(at + \frac{bt^2}{2}\right)} \dots (1.2)$$

The inventory without decay  $I_w(t)$  at time 't' is given as

$$\frac{d}{dt}I_{w}(t) = -d(P)e^{-\alpha t} = -d(P)(1-\alpha t) \qquad \dots (1.3)$$

Solution of equation (1.3) is given by

$$I_w(t) = -d(P)\left(t - \frac{\alpha t^2}{2}\right) + I(0)$$
 ...(1.4)

Stock loss due to decay Z(t) at time t is given by

$$Z(t) = I_{w}(t) - I(t)$$
  
=  $I(t) \left[ at + \frac{bt^{2}}{2} \right] + d(P) \left[ t + \frac{(a - \alpha)t^{2}}{2} + \frac{bt^{3}}{6} \right] - d(P) \left( t - \frac{\alpha t^{2}}{2} \right)$   
... (1.5)

Depletion of stock in the interval  $(0,T_1)$  is due to decay and demand and in interval  $(T_1,T)$ excess demand is backlogged as well.

The loss of inventory due to deterioration in the cycle T is given by equation (1.5) by putting  $t = T_1, I(t) = 0.$ 

$$Z(T_1) = d(P) \left[ T_1 + \frac{(a-\alpha)T_1^2}{2} + \frac{bT_1^3}{6} \right] - d(P) \left( T_1 - \frac{\alpha T_1^2}{2} \right). \quad \dots (1.6)$$

Backlogged demand within the cycle is given by

$$B(T_{1}) = d(P) \int_{T_{1}}^{T} e^{-\alpha t} dt = d(P) \int_{T_{1}}^{T} (1 - \alpha t) dt$$
  
=  $d(P) \left[ (T - T_{1}) - \frac{\alpha (T^{2} - T_{1}^{2})}{2} \right].$  ...(1.7)

The order quantity  $Q_T$  is given as

$$Q_{T} = d(P)(T - T_{1}) + d(P)\left[T_{1} + \frac{(a - \alpha)T_{1}^{2}}{2} + \frac{bT_{1}^{3}}{6}\right] - \frac{d(P)\alpha T^{2}}{2}$$
$$Q_{T_{1}} = d(P)\left[T_{1} + \frac{(a - \alpha)T_{1}^{2}}{2} + \frac{bT_{1}^{3}}{6}\right] - \frac{d(P)\alpha T_{1}^{2}}{2}.$$
...(1.8)

Also noting that  $I(0) = Q_{T_1}$ So by equation (1.2), we get

$$I(t) = -d(P)e^{-\left(at + \frac{bt^2}{2}\right)} \left[t + \frac{(a - \alpha)t^2}{2} + \frac{bt^3}{6}\right]$$

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$$+d(P)\left[T_{1}+\frac{(a-\alpha)T_{1}^{2}}{2}-\frac{\alpha T_{1}^{2}}{2}+\frac{bT_{1}^{3}}{6}\right]e^{-\left(at+\frac{bt^{2}}{2}\right)}.$$
 ...(1.9)

Now total cost per cycle is given by

 $C_1(T, T_I, P)$  = Order cost + Purchase cost + Holding cost + Shortage cost

$$\begin{split} &= \mathrm{K} + \mathrm{CQ}_{\mathrm{T}} + \int_{0}^{\mathrm{T}_{1}} (\mathrm{h} + \mathrm{\phi} t) \mathrm{I}(t) \mathrm{d} t + \mathrm{S} \int_{0}^{\mathrm{T} - \mathrm{T}_{1}} \mathrm{d}(\mathrm{P}) \mathrm{e}^{-\alpha t} t \mathrm{d} t \\ &= \mathrm{K} + \mathrm{Cd}(\mathrm{P}) \bigg[ \mathrm{T} + \frac{(\mathrm{a} - \alpha) \mathrm{T}_{1}^{2}}{2} + \frac{\mathrm{b} \mathrm{T}_{1}^{3}}{6} - \frac{\alpha \mathrm{T}^{2}}{2} \bigg] \\ &+ \int_{0}^{\mathrm{T}_{1}} (\mathrm{h} + \mathrm{\phi} t) \bigg\{ \mathrm{d}(\mathrm{P}) \mathrm{e}^{-\left(\mathrm{a} t + \frac{\mathrm{b} t^{2}}{2}\right)} \\ &\left( \mathrm{T}_{1} + \frac{(\mathrm{a} - \alpha) \mathrm{T}_{1}^{2}}{2} + \frac{\mathrm{b} \mathrm{T}_{1}^{3}}{6} - \mathrm{t} - \frac{(\mathrm{a} - \alpha) t^{2}}{2} - \frac{\mathrm{b} t^{3}}{6} - \frac{\alpha \mathrm{T}_{1}^{2}}{2} \right) \bigg\} \\ &+ \mathrm{S} \int_{0}^{\mathrm{T} - \mathrm{T}_{1}} \mathrm{d}(\mathrm{P}) \bigg[ \mathrm{t} - \frac{\alpha t^{2}}{2} \bigg] \mathrm{d} t \\ &= \mathrm{K} + \mathrm{Cd}(\mathrm{P}) \bigg[ \mathrm{T} + \frac{(\mathrm{a} - \alpha) \mathrm{T}_{1}^{2}}{2} + \frac{\mathrm{b} \mathrm{T}_{1}^{3}}{6} - \frac{\alpha \mathrm{T}^{2}}{2} \bigg] \\ &+ \mathrm{hd}(\mathrm{P}) \bigg( \frac{\mathrm{T}_{1}^{2}}{2} + \frac{\mathrm{a} \mathrm{T}_{1}^{3}}{3} - \frac{5\alpha \mathrm{T}_{1}^{3}}{6} - \frac{\mathrm{a}^{2} \mathrm{T}_{1}^{4}}{8} + \frac{\mathrm{b} \mathrm{T}_{1}^{4}}{12} + \frac{3\mathrm{a} \alpha \mathrm{T}_{1}^{4}}{8} \\ &+ \frac{7\mathrm{b} \alpha \mathrm{T}_{1}^{5}}{60} - \frac{\mathrm{b}^{2} \mathrm{T}_{1}^{6}}{72} \bigg) + \mathrm{\phi} \mathrm{d}(\mathrm{P}) \bigg( \frac{\mathrm{T}_{1}^{3}}{6} + \frac{\mathrm{a} \mathrm{T}_{1}^{4}}{24} - \frac{3\alpha \mathrm{T}_{1}^{4}}{8} - \frac{\mathrm{a}^{2} \mathrm{T}_{1}^{5}}{15} \\ &- \frac{7\mathrm{b} \mathrm{T}_{1}^{5}}{120} + \frac{7\mathrm{a} \alpha \mathrm{T}_{1}^{5}}{30} + \frac{\mathrm{a} \mathrm{b} \mathrm{T}_{1}^{5}}{12} - \frac{7\mathrm{a} \mathrm{b} \mathrm{T}_{1}^{6}}{144} + \frac{\mathrm{b} \mathrm{c} \mathrm{T}_{1}^{6}}{12} - \frac{\mathrm{b}^{2} \mathrm{T}_{1}^{7}}{112} \bigg) \\ &+ \mathrm{Sd}(\mathrm{P}) \bigg[ \frac{\left(\mathrm{T} - \mathrm{T}_{1}\right)^{2}}{2} - \frac{\alpha (\mathrm{T} - \mathrm{T}_{1})^{3}}{6}} \bigg]. \end{split}$$

Total cost  $C_1(T, T_1, P)$  per unit time is

$$C(T, T_{1}, P) = \frac{C_{1}(T, T_{1}, P)}{T}$$

$$\Rightarrow C(T, T_{1}, P) = \frac{K}{T} + Cd(P) + \frac{Cd(P)(a - \alpha)T}{2} \cdot \frac{T_{1}^{2}}{T^{2}}$$

$$+ \frac{bCd(P)T^{2}}{6} \cdot \frac{T_{1}^{3}}{T^{3}} - \frac{Cd(P)\alpha T}{2} \cdot \frac{T_{1}^{2}}{T^{2}} + \frac{bd(P)\alpha T}{2} \cdot \frac{T_{1}^{2}}{T^{2}}$$

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$$+\frac{ahd(P)T^{2}}{6}\cdot\frac{T_{1}^{3}}{T^{3}}-\frac{5hd(P)\alpha T^{2}}{6}\cdot\frac{T_{1}^{3}}{T^{3}}\\-\frac{a^{2}hd(P)T^{3}}{8}\cdot\frac{T_{1}^{4}}{T^{4}}+\frac{bhd(P)T^{3}}{12}\cdot\frac{T_{1}^{4}}{T^{4}}$$

$$+ \frac{3ahd(P)\alpha T^{3}}{8} \cdot \frac{T_{1}^{4}}{T^{4}} + \frac{7bhd(P)\alpha T^{4}}{60} \cdot \frac{T_{1}^{5}}{T^{5}} - \frac{b^{2}hd(P)T^{5}}{72} \cdot \frac{T_{1}^{6}}{T^{6}} \\ + \frac{\phi d(P)T^{2}}{6} \cdot \frac{T_{1}^{3}}{T^{3}} + \frac{a\phi d(P)T^{3}}{24} \cdot \frac{T_{1}^{4}}{T^{4}} - \frac{3\phi d(P)\alpha T^{3}}{8} \cdot \frac{T_{1}^{4}}{T^{4}} \\ - \frac{a^{2}\phi d(P)t^{4}}{15} \cdot \frac{T_{1}^{5}}{T^{5}} - \frac{7b\phi d(P)T^{4}}{120} \cdot \frac{T_{1}^{5}}{T^{5}} + \frac{7a\phi d(P)\alpha T^{4}}{30} \cdot \frac{T_{1}^{5}}{T^{5}} \\ + \frac{ab\phi d(P)T^{4}}{12} \cdot \frac{T_{1}^{5}}{T^{5}} - \frac{7ab\phi d(P)T^{5}}{144} \cdot \frac{T_{1}^{6}}{T^{6}} + \frac{b\phi d(P)\alpha T^{5}}{12} \cdot \frac{T_{1}^{6}}{T^{6}} \\ \frac{b^{2}\phi d(P)T^{6}}{112} \cdot \frac{T_{1}^{7}}{T^{7}} + \frac{Sd(P)T}{2} \left(1 - \frac{T_{1}}{T}\right)^{2} - \frac{Sd(P)\alpha T^{2}}{6} \left(1 - \frac{T_{1}}{T}\right)^{3}.$$

Let  $\frac{\mathbf{I}_1}{T} = \eta$  be the fraction of cycle when there is no excess demand. Now cost per unit time can pressed as a function of T. n and P. So, we have

be expressed as a function of T, 
$$\eta$$
 and P. So, we have

$$\begin{split} C(T,\eta,P) &= \frac{K}{T} + Cd(P) + \frac{Cd(P)(a-\alpha)T}{2} \cdot \eta^{2} \\ &+ \frac{bCd(P)T^{2}}{6} \cdot \eta^{3} - \frac{Cd(P)\alpha T}{2} \cdot \eta^{2} + \frac{hd(P)T}{2} \cdot \eta^{2} \\ &+ \frac{ahd(P)T^{2}}{6} \eta^{3} - \frac{5hd(P)\alpha T^{2}}{6} \cdot \eta^{3} - \frac{a^{2}hd(P)T^{3}}{8} \cdot \eta^{4} \\ &+ \frac{bhd(P)T^{3}}{12} \cdot \eta^{4} + \frac{bhd(P)T^{3}}{12} \cdot \eta^{4} + \frac{3ahd(P)\alpha T^{3}}{8} \cdot \eta^{4} \\ &+ \frac{7bhd(P)\alpha T^{4}}{60} \cdot \eta^{5} - \frac{b^{2}hd(P)T^{5}}{72} \cdot \eta^{6} + \frac{\phi d(P)T^{2}}{6} \cdot \eta^{3} \\ &+ \frac{a\phi d(P)T^{3}}{24} \cdot \eta^{4} - \frac{3\phi d(P)\alpha T^{3}}{8} \cdot \eta^{4} - \frac{a^{2}\phi d(P)T^{4}}{15} \cdot \eta^{5} \\ &- \frac{7b\phi d(P)T^{4}}{120} \cdot \eta^{5} + \frac{7a\phi d(P)\alpha T^{4}}{30} \cdot \eta^{5} \\ &+ \frac{ab\phi d(P)T^{4}}{12} \cdot \eta^{5} - \frac{7ab\phi d(P)T^{5}}{144} \cdot \eta^{6} + \frac{b\phi d(P)\alpha T^{5}}{12} \cdot \eta^{6} \\ &- \frac{b^{2}\phi d(P)T^{6}}{112} \cdot \eta^{7} + \frac{Sd(P)T}{2}(1-\eta)^{2} - \frac{Sd(P)\alpha T^{2}}{6}(1-\eta)^{3}. \end{split}$$

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...(1.10)

Now we consider the following boxes:

#### Box I.

For fixed unit selling price 'P' the total cost  $C(T, \eta, P)$  will be minimum for minimum value of T and  $\eta$ .

For minimum value of T, putting  $\frac{\partial C}{\partial T} = 0$ , we get  $-\frac{K}{T} + \frac{Cd(P)(a-\alpha)}{2} \cdot \eta^{2} + \frac{bCd(P)T}{3} \cdot \eta^{3} - \frac{Cd(P)\alpha}{2} \cdot \eta^{2} + \frac{hd(P)T}{2} \cdot \eta^{2} + \frac{ahd(P)T}{3} \eta^{3} - \frac{5hd(P)\alpha T}{3} \cdot \eta^{3} - \frac{3a^{2}hd(P)T^{2}}{8} \cdot \eta^{4} + \frac{bhd(P)T^{2}}{4} \cdot \eta^{4} + \frac{9ahd(P)\alpha T^{2}}{8} \cdot \eta^{4} + \frac{7bhd(P)\alpha T^{3}}{15} \cdot \eta^{5} - \frac{5b^{2}hd(P)T^{4}}{72} \cdot \eta^{6} - \frac{7b\phi d(P)T^{3}}{30} \cdot \eta^{5} + \frac{\phi d(P)T}{3} \cdot \eta^{3} + \frac{a\phi d(P)T^{2}}{8} \cdot \eta^{4} - \frac{9\phi d(P)\alpha T^{2}}{8} \cdot \eta^{4} + \frac{-4a^{2}\phi d(P)T^{3}}{15} \cdot \eta^{5} + \frac{14a\phi d(P)\alpha T^{3}}{15} \cdot \eta^{5} + \frac{ab\phi d(P)T^{3}}{3} \cdot \eta^{5} + \frac{35ab\phi d(P)T^{4}}{144} \cdot \eta^{6} + \frac{5b\phi d(P)\alpha T^{4}}{12} \cdot \eta^{6} - \frac{3b^{2}\phi d(P)T^{5}}{56} \cdot \eta^{7} + \frac{Sd(P)}{2}(1-\eta)^{2} - \frac{Sd(P)\alpha T}{3}(1-\eta)^{3} = 0. \quad ...(1.11)$ 

Equation (1.11) can be solved for T for given values of K, h, C,  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\eta$ , a, b and S. Now for minimum value of  $\eta$ , putting  $\frac{\partial C}{\partial n} = 0$ , one can get by equation (1.10).

$$Cd(P)(a-\alpha)T\eta + \frac{bCd(P)T^{2}}{2} \cdot \eta^{2} - Cd(P)\alpha T \cdot \eta + hd(P)T\eta \\ + \frac{ahd(P)T^{2}\eta^{2}}{2} - \frac{5hd(P)\alpha T^{2}}{2} \cdot \eta^{2} - \frac{a^{2}hd(P)T^{3}}{2} \cdot \eta^{3} \\ + \frac{bhd(P)T^{3}}{3} \cdot \eta^{3} + \frac{3ahd(P)\alpha T^{3}}{2} \cdot \eta^{3} + \frac{7bhd(P)\alpha T^{4}}{12} \cdot \eta^{4} \\ - \frac{b^{2}hd(P)T^{5}}{12} \cdot \eta^{5} + \frac{\phi d(P)T^{2}}{2} \cdot \eta^{2} + \frac{a\phi d(P)T^{3}}{6} \cdot \eta^{3} \\ - \frac{3\phi d(P)\alpha T^{3}}{2} \cdot \eta^{3} - \frac{a^{2}\phi d(P)T^{4}}{3} \cdot \eta^{4} - \frac{7b\phi d(P)T^{4}}{24} \cdot \eta^{4}$$

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$$+\frac{7a\phi d(P)\alpha T^{4}}{6}\cdot\eta^{4}+\frac{5ab\phi d(P)T^{4}}{12}\cdot\eta^{4}-\frac{7ab\phi d(P)T^{5}}{24}\cdot\eta^{5}$$
$$+\frac{b\phi d(P)\alpha T^{5}}{2}\cdot\eta^{5}-\frac{b^{2}\phi d(P)T^{6}}{16}\cdot\eta^{6}-\frac{Sd(P)T}{2}(1-\eta)$$

$$+\frac{\mathrm{Sd}(P)\alpha T^{2}}{2}(1-\eta)^{2}=0. \qquad \dots (1.12)$$

For the values of T and  $\eta$  obtained from equation (1.11) and (1.12) respectively, it can be shown that

$$\frac{\partial^2 C}{\partial T^2} > 0 \text{ and } \frac{\partial^2 C}{\partial T^2} > 0.$$
 ...(1.13)

Box II.

For optimal price decision, consider the profit rate function for a fixed period length i.e. T = constant

$$f(\mathbf{T}, \mathbf{t}, \boldsymbol{\eta}, \mathbf{P}) = Pd(\mathbf{P})\ell^{-\alpha t} - C(\mathbf{T}, \boldsymbol{\eta}, \mathbf{P}). \qquad \dots (1.14)$$
  
Differentiating equation (1.14) with respect to P and equating to zero, we get

$$d(\mathbf{P})\ell^{-\alpha t} + \mathbf{P}d'(\mathbf{P})\ell^{-\alpha t} - \frac{\partial \mathbf{C}(\mathbf{T},\boldsymbol{\eta},\mathbf{P})}{\partial \mathbf{P}} = 0. \qquad \dots (1.15)$$

Also we have from equation (1.10)

$$\frac{\partial C(T,\eta,P)}{\partial P} = d'(P) \left[ C + \frac{c(a-\alpha)T}{2} \cdot \eta^2 + \frac{bcT^2}{6} \cdot \eta^3 - \frac{c\alpha T}{2} \cdot \eta^2 + \frac{hT}{2} \cdot \eta^2 + \frac{ahT^2}{6} \cdot \eta^3 - \frac{5h\alpha T^2}{6} \cdot \eta^3 - \frac{a^2hT^3}{8} \cdot \eta^4 + \frac{bhT^3}{12} \cdot \eta^4 + \frac{3ah\alpha T^3}{8} \cdot \eta^4 + \frac{7bh\alpha T^4}{60} \cdot \eta^5 - \frac{b^2hT^5}{72} \cdot \eta^6 + \frac{\phi T^2}{6} \cdot \eta^3 + \frac{a\phi T^3}{24} \cdot \eta^4 - \frac{3\phi\alpha T^3}{8} \cdot \eta^4 - \frac{a^2\phi T^4}{15} \cdot \eta^5 - \frac{7b\phi T^4}{120} \cdot \eta^5 + \frac{7a\phi\alpha T^4}{30} \cdot \eta^5 + \frac{ab\phi T^4}{12} \cdot \eta^5 - \frac{7ab\phi T^5}{144} \cdot \eta^6 + \frac{b\phi\alpha T^5}{12} \cdot \eta^6 - \frac{b^2\phi T^6}{112} \cdot \eta^7 + \frac{ST}{2}(1-\eta)^2 - \frac{S\alpha T^2}{6}(1-\eta)^3 \right]$$

Substituting the value of  $\frac{\partial C(T,\eta,P)}{\partial P}$  in equation (1.15) and simplifying, we get

$$P = \ell^{\alpha t} \left[ c + \frac{c(a-\alpha)T}{2} \eta^2 + \frac{bcT^2}{6} \cdot \eta^2 - \frac{c\alpha T}{2} \eta^2 + \frac{hT^2}{2} \eta^2 + \frac{hT^2}{6} \eta^3 - \frac{5h\alpha T^2}{6} \eta^3 - \frac{a^2hT^3}{8} \eta^4 + \frac{bhT^3}{12} \eta^4 \right]$$

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$$\begin{aligned} &+\frac{3ah\alpha T^{3}}{8}\eta^{4}+\frac{7bh\alpha T^{4}}{60}\eta^{5}-\frac{b^{2}hT^{5}}{72}\eta^{6}+\frac{\phi T^{2}}{6}\eta^{3}+\frac{a\phi T^{3}}{24}\eta^{4} \\ &-\frac{3\phi\alpha T^{3}}{8}\eta^{4}-\frac{a^{2}\phi T^{4}}{15}\eta^{5}-\frac{7b\phi T^{4}}{120}\eta^{5}+\frac{7a\phi\alpha T^{4}}{30}\eta^{5} \\ &+\frac{ab\phi T^{4}}{12}\eta^{5}-\frac{7ab\phi T^{5}}{144}\eta^{6}+\frac{b\phi\alpha T^{5}}{12}\eta^{6}-\frac{b^{2}\phi T^{6}}{112}\eta^{7} \\ &+\frac{ST}{2}(1-\eta)^{2}-\frac{S\alpha T^{2}}{6}(1-\eta)^{3}\left]-\frac{d(P)}{d'(P)}.\qquad ...(1.16) \end{aligned}$$
Also P > 0 and  $\frac{\partial^{2}f(T,\eta,P)}{\partial P^{2}} < 0$  giving P, as the optimal price.

#### CONCLUSION

In this paper, an inventory model for decaying items has been studied. It is assumed that the demand rate is time dependent and a ramp type pattern of three branches has been used. The model is more realistic as Weibull distributed deterioration rate is taken into account. The whole concept of this model is illustrated with a numerical example and sensitivity analysis is also performed.

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